C13

Roles of Groove and Waviness in Control of Stiction and Slip Nucleation

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1. Introduction

The law of static friction (stiction) is generally known as Amontons' law, which indicates that the static friction coefficient is independent of the object shape and implies that stiction cannot be controlled by designing the object shape. However, the phenomenological explanation of Amontons' law, which considers the real contact area, implicitly assumes uniformity of the stress field, and the law breaks down otherwise[1, 2]. Recent studies reveal that the static friction coefficient depends on object shape due to the quasi-static local slip (nucleation) in some macroscopic systems[2-4]. They employ systems where the lateral side of the object is loaded, and the slip nucleation starts from the lateral edge. The nucleation has a critical length that positively correlates with the static friction coefficient, and the slip region accelerates rapidly after the nucleation reaches the critical length. A recent study reveals that the critical length of nucleation and the static friction coefficient decrease by forming grooves due to decreased stiffness[4].

However, the previous results are limited to systems with side load, and it is unclear whether they are applicable in general systems with uniform shear loads. Moreover, there is no guiding principle to keep high friction with grooves, which is required for some systems such as tires. Therefore, we investigate the stiction of objects with grooves under uniform shear load and how it can be used to control friction.

2. Numerical calculation with 3D finite element method

2.1 Setting of system

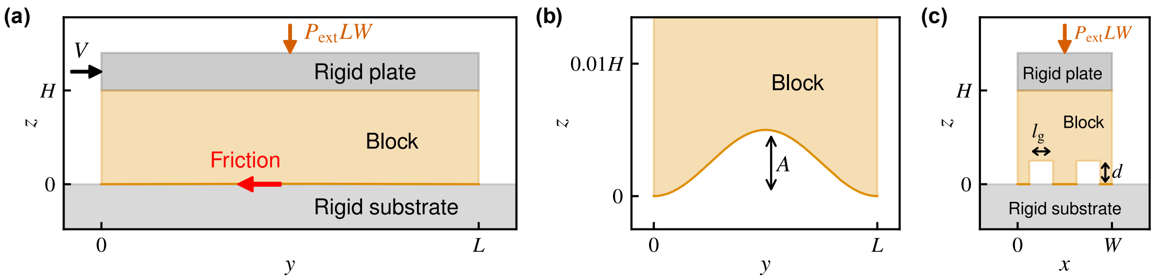

Fig. 1 3D grooved block with waviness for $A/H = 0.005$, $n_x = 2$, $\phi = 0.5$, and $d/H = 0.25$. **a**, **b** Cross section perpendicular to x-direction at $y = 0.5W$. **b** Friction surface magnified in z direction. **c** Cross section perpendicular to y-direction.

We consider a 3D grooved block with height H and Kelvin-Voigt viscoelasticity of Young's modulus E, Poisson's ratio ν, and viscosity coefficient η on a rigid substrate, as shown in Fig. 1a. The top rigid plate parallel to the substrate drives the block with slow y-direction velocity V. The external pressure $P_{\rm ext}$ is loaded on the top plate. We consider periodic boundaries with periods W and L in the x and y directions, respectively. Under this uniform shear set-up, Amontons' law is expected to hold macroscopically if the frictional strength is also uniform, where only spatially confined nucleation occurs. Thus, we introduce waviness, as shown in Fig. 1b, to create a heterogeneous pressure and frictional strength, which induces localized nucleation[5]. The bottom height is given by $A[1 - \cos(2\pi y/L)]/2$ with small maximum amplitude A. Here, the friction surface is in full contact with the substrate when the external normal load is applied. We consider grooves on the friction surface parallel to the y-direction, where the number, width, and depth of grooves are n_x, $l_{\rm g}$, and d, respectively, as shown in Fig. 1c. The decreasing fraction of the contact area is given by $\phi = n_x l_{\rm g}/W$, where $\phi = 0$ corresponds to a block without grooves.

The friction between the block bottom and substrate obeys the Amontons-Coulomb law locally, where the local frictional stress is given by $\mu(v(x,y,t))p(x,y,t)$ with time t, the local friction coefficient $\mu(v)$, the slip velocity v, and bottom pressure p[2-4]. During the slip phase, the local friction coefficient $\mu(v)$ decreases from $\mu_{\rm S}$ to $\mu_{\rm K}$ by increasing v in $v \le v_{\rm c}$ (the weakening phase) and stays at $\mu_{\rm K}$ in $v > v_{\rm c}$ (the residual phase), where $\mu_{\rm S}$ and $\mu_{\rm K}$ are local static and kinetic friction coefficients, respectively, and $v_{\rm c}$ is a characteristic velocity. Here, the driving velocity V satisfies $V \ll v_{\rm c}$.

2.2 Results

Here, we show results for $W/H = 1$, $L/H = 4$, $n_x = 2$, $d/H = 0.25$, and $P_{\rm ext}/E = 0.003$. Figure 2a shows the friction force $F_{\rm T}$ normalized by the normal load $F_{\rm N} = P_{\rm ext}LW$ in the considered region against the rigid plate displacement $U = Vt$ for $\phi = 0.5$. The block exhibits periodic stick-slip motion, and the maximum value of $F_{\rm T}/F_{\rm N}$ is defined as the macroscopic static

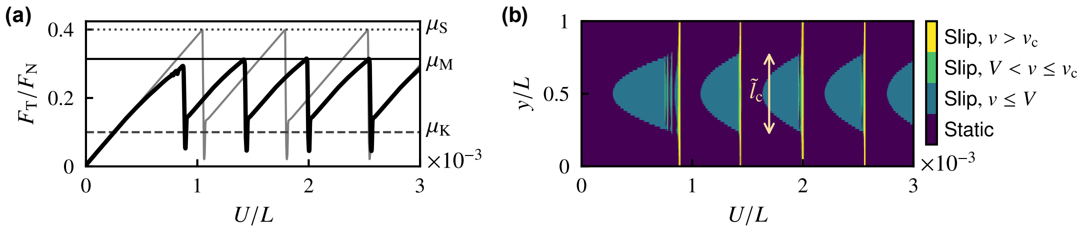

Fig. 2 **a** Friction force F_T against U for $\phi = 0.5$. Gray and black lines represent results for $A = 0$ and $A/H = 0.005$, respectively. The solid horizontal line represents μ_M for $A/H = 0.005$. Dotted and dashed lines represent μ_S and μ_K, respectively. **b** Spatial distribution of slip velocity v at $x/W = 0.5$ against U for $\phi = 0.5$ and $A/H = 0.005$. Blue, yellow-green, and yellow areas represent slip phases with $v \leq V$, $V < v \leq v_c$, and $v > v_c$, respectively.

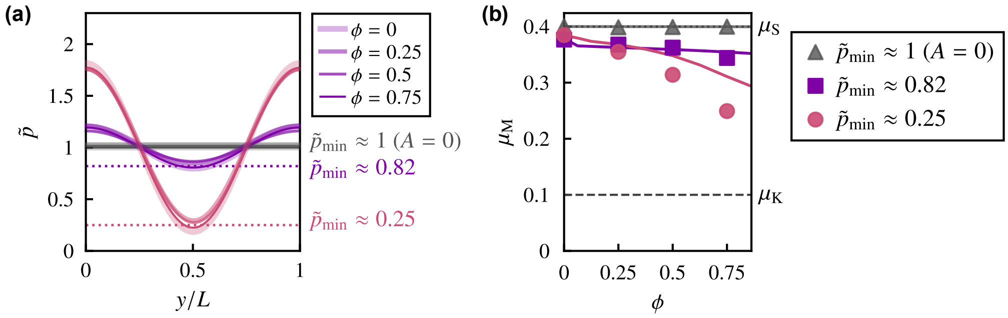

Fig. 3 **a** Spatial distribution of $\tilde{p}$ at $x/W = 0.5$ and $U/L = 3 \times 10^{-3}$ for different values of ϕ and A. Dotted lines represent $\tilde{p}_{min}$. **b** Static friction coefficient μ_M against ϕ. Dotted and dashed lines represent μ_S and μ_K, respectively. Symbols and solid lines represent the results of 3D FEM and 1D model, respectively.

friction coefficient μ_M. Although μ_M is consistent with μ_S for $A = 0$, μ_M becomes smaller than μ_S for $A/H = 0.005$. Figure 2b shows the spatial distribution of slip velocity v at $x/W = 0.5$ against U for $\phi = 0.5$ and $A/H = 0.005$. Slip nucleation is initiated from the center, and the slip region accelerates rapidly after it reaches the critical length $\tilde{l}_c$ and propagates across the entire interface.

Table 1 Values of A/H used in Fig. 2

	$\tilde{p}_{min} \approx 1$	$\tilde{p}_{min} \approx 0.82$	$\tilde{p}_{min} \approx 0.25$
$\phi = 0$	0	0.001	0.004
$\phi = 0.25$	0	0.001	0.004
$\phi = 0.5$	0	0.001	0.005
$\phi = 0.75$	0	0.002	0.008

Figure 3a, b shows the distribution of normalized bottom pressure $\tilde{p} = p(1-\phi)/P_{ext}$ at $x/W = 0.5$ and $U/L = 3 \times 10^{-3}$ and the static friction coefficient μ_M, respectively, for different values of ϕ and A. Note that $\tilde{p}$ is approximately uniform in x direction and stationary. We select the values of A depending on ϕ, as shown in Table 1, to compare the results in the approximately same profiles of $\tilde{p}$, as shown in Fig. 3a. The minimum value of $\tilde{p}$ is denoted by $\tilde{p}_{min}$, which is a decreasing function of A/H depending on ϕ.

Figure 3b shows that μ_M is an increasing function of $\tilde{p}_{min}$. The shear stress increases approximately uniformly, and nucleation is initiated earlier for smaller $\tilde{p}_{min}$ (larger A) because slip occurs at the region where the shear stress reaches the frictional strength $\mu_S p$. The nucleation also reaches the critical length $\tilde{l}_c$ with a lower shear stress level for smaller $\tilde{p}_{min}$, which leads to the decrease in μ_M. For $\tilde{p}_{min} \approx 1$ ($A = 0$), μ_M remains at μ_S with any values of ϕ, where only spatially confined nucleation occurs.

The static friction coefficient μ_M should be the same for the same profile of $\tilde{p}$ if the critical lengths of nucleation $\tilde{l}_c$ were the same. However, Fig. 3b shows that μ_M is a decreasing function of ϕ, which indicates the critical length $\tilde{l}_c$ also decreases. This result is consistent with the previous study with the side load system[4].

3. 1D model

To understand the results of 3D FEM, we introduce and analyze a 1D model, as shown in Fig. 4. The 1D viscoelastic object is driven by the shear spring. We consider the y-direction displacement along y direction of the 1D viscoelastic object. We give the heterogeneous pressure, following Fig. 3a. The effect of grooves is incorporated in the effective elastic modulus, viscous coefficient, and density[4], where a larger groove corresponds to lower elasticity, viscosity, and density. We calculate the time evolution of the 1D model. The model exhibits the stick-slip motion and slip nucleation.

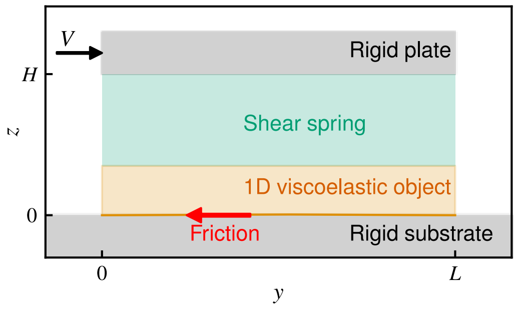

Fig. 4 Schematic of 1D model

The solid lines in Fig. 3b represent the static friction coefficient μ_M obtained by the calculation of 1D model. The figure shows that the 1D model reproduces the results of 3D FEM, qualitatively. The results can be explained by the theories considering viscosity[4] or elasticity[5]. The common point in both theories is that forming grooves decreases the stiffness, which shortens the critical length of nucleation and reduces μ_M.

4. Conclusion

This study shows that slip nucleation and static friction coefficient can be controlled even in systems under uniform shear by designing waviness and grooves. The analyses with the 3D FEM and the 1D model present that the static friction coefficient is scaled by the heterogeneous pressure profile induced by the waviness, which determines how the nucleation initiates and grows, and the groove size, which decreases the stiffness of the system and shortens the critical length of nucleation. The new guiding principle of the design is that reducing waviness with or without grooves is needed for high stiction, and some waviness and large grooves are needed for low stiction.

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