Modeling of starved hydrodynamic lubrication considering flow separation and surface tension

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1. Introduction

In simulations of starved lubrication, the Elrod-Adams (E-A) model is widely used, where surface tension is typically neglected. However, experimental studies [1,2] have demonstrated that the backflow of lubricant induced by surface tension significantly affects the film distribution near the contact area. The characteristic butterfly-shaped film distribution observed experimentally is not predicted by the E-A model. In this study, a two-dimensional model that incorporates surface tension and accounts for separation flow due to film rupture near the outlet of the contact area is proposed. The new model is solved using the CIP-CSL2 method [3], a high-accuracy numerical scheme.

2. Modelling

Figure 1 shows the schematic of the two-dimensional starved model. In this model, the upper surface is stationary, while the lower surface moves at a constant sliding velocity u. Lubricant is distributed between the upper and lower surfaces with an initial film thickness of h_s. Table 1 shows additional simulation parameters. The flow of lubricant in all domains can be characterized by the following equation

$$\frac{\partial}{\partial x}\left(-\frac{h^3}{12\mu}\frac{\partial p}{\partial x}+\frac{\theta h}{2}(v_x+u)\right)+\frac{\partial}{\partial y}\left(-\frac{h^3}{12\mu}\frac{\partial p}{\partial y}+\frac{\theta h}{2}v_y\right)+\frac{\partial(\theta h)}{\partial t}=0 \tag{1}$$

where saturation variable θ is the ratio of the film thickness to the local gap h, and h can be represented as

$$h = h_0 + 4\delta x^2/l^2 \tag{2}$$

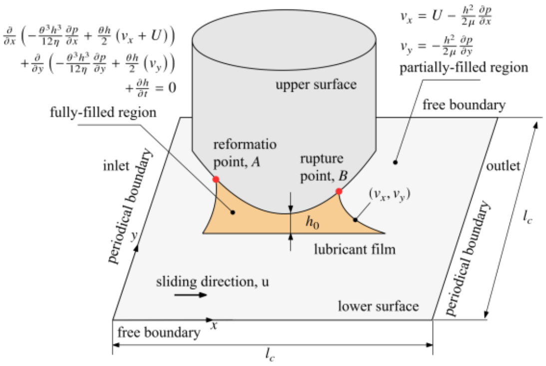

Fig. 1 Schematic of the starvation model.

Table 1 Simulation parameters

Parameters	**Value**
Calculation zone, l	1 mm
Pin height, σ	32 μm
Minimum film thickness, h_0	1 μm
Initial lubricant supply, h_s	1.5 μm
Viscosity, μ	0.01 Pas
Surface tension, γ	0.07 N/m
Sliding speed, u	1 m/s
Mesh size	80
CFL number	0.25

v_x and v_y represent the flow velocities of lubricant on the free surface in the x- and y-axis directions. As the lubricant flows, the lubrication region is divided into two parts. In the fully-filled region, the hydrodynamic pressure is generated, and it has

$$\begin{cases}\theta = 1\\ p > max\{p_s, p_{cav}\}\\ v_x = 0; v_y = 0\end{cases} \tag{3}$$

In the partially-filled region, lubricant separates from the upper surface, with the flow being controlled by the following equations (4), in which p_{cav} is the cavitation pressure. p_s is the pressure at the free surface of lubricant.

$$\begin{cases}\theta < 1\\ p = max\{p_s, p_{cav}\}\\ v_x = u - \frac{(h\theta)^2}{2\mu}\frac{\partial p}{\partial x}\\ v_y = -\frac{(h\theta)^2}{2\mu}\frac{\partial p}{\partial y}\end{cases} \qquad \text{where} \qquad p_s = p_{amb} - \gamma\left(\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}\right) \tag{4}$$

P_{amb} represents the ambient pressure, and γ is the surface tension. To provide a unified description of the lubricant flow on the free surface, a governing function is proposed, which can be written as:

$$v_x = \left(\frac{2}{max(2\theta,1)} - 1\right)\left(u - \frac{(\theta h)^2}{2\mu}\frac{\partial p}{\partial x}\right);\ v_y = \left(\frac{2}{max(2\theta,1)} - 1\right)\left(-\frac{(\theta h)^2}{2\mu}\frac{\partial p}{\partial y}\right) \tag{5}$$

Considering the solution accuracy, the CIP-CSL2 method combined with the fractional step technique is employed to solve the aforementioned equations.

3. Results and discussion

Figure 2 compares the simulation results of the E-A model and the proposed model. At $T = 1$, the proposed model predicts a larger contact area than the E-A model. A comparison of the film and pressure distributions along the x-axis at the contact center reveals that the proposed model predicts a smaller film thickness in the partially-filled region, with a discontinuous distribution observed near the outlet. In terms of pressure, the proposed model predicts a larger hydrodynamic pressure compared to the E-A model. The reason for this result is that the generalized E-A model underestimates the mass transport rate within the partially-filled region [4].

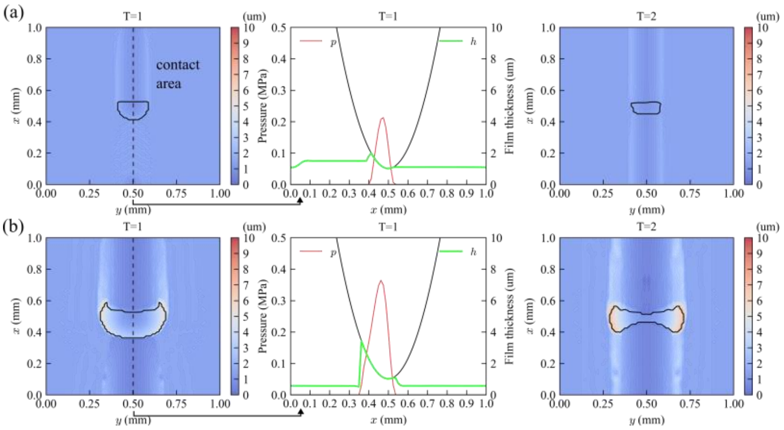

Fig. 2 Comparison of simulation results of (a) the Elrod-Adams model and (b) the proposed model.

At T = 2, the film distribution in the E-A model adopts a rectangular shape, indicating that the inlet meniscus boundary is very close to the center of the contact area, with no significant lubricant accumulation near the contact area. In contrast, the proposed model predicts a larger contact region, with oil-rich regions on both sides of the contact area. Under the influence of surface tension, the film distribution takes on a butterfly shape, with backflow occurring near the inlet, consistent with the phenomena observed in experiments.

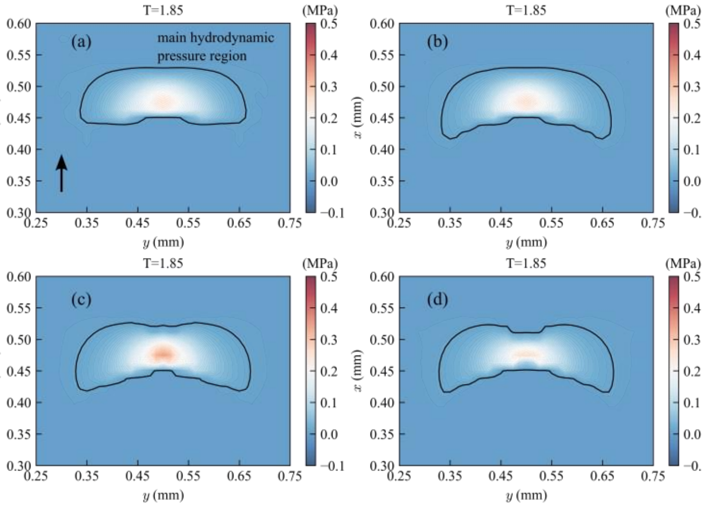

Fig. 3 Effect of surface tension on hydrodynamic pressure: (a) $\gamma = 0.01$ N/m, (b) $\gamma = 0.03$ N/m, (c) $\gamma = 0.07$ N/m, (d) $\gamma = 0.1$ N/m.

Figure 3 illustrates the impact of surface tension on hydrodynamic pressure when starvation occurs. As surface tension increases from 0.01 to 0.07 N/m, the backflow near the inlet region increases. This is evident from the main hydrodynamic pressure regions shifting towards a butterfly shape, causing an increase in the hydrodynamic pressure. It indicates that higher surface tension contributes to an enhanced film thickness in the contact area. However, when surface tension reaches 0.1 N/m, the hydrodynamic pressure in the contact area decreases instead. This decrease is attributed to excessively high surface tension causing the lubricant film to rupture or separate more easily at the outlet region [5]. Therefore, an appropriate level of surface tension would be essential for improving the starved lubrication.

4. Conclusions

This study proposed a new two-dimensional model that incorporates surface tension at the free surface of lubricant and accounts for separation flow near the outlet region. Compared to the generalized E-A model, the proposed model successfully replicates the classic butterfly-shaped film distribution for starved lubrication. Increasing surface tension enhances backflow at the inlet zone and raises hydrodynamic pressure. However, excessively high surface tension accelerates film rupture and separation at the outlet region.

References

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