

# Flow separation and surface tension considered model for starved hydrodynamic lubrication problems

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## 1. Introduction

The experiment tests [1,2] of starved lubrication show that oil film presents a butterfly-shaped distribution, and the inlet film thickness is affected by the backflow from the oil reservoirs on both sides of the contact area. However, the commonly used Elrod-Adams (E-A) models fail to predict that film behavior, and backflow effects are also ignored. In this study, a two-dimensional model that incorporates surface tension and accounts for separation flow due to film rupture near the outlet of the contact area was proposed. The classic butterfly-shaped oil lubrication is reproduced. The results indicate that the lubricant replenishment improves with increasing surface tension but decreases as viscosity increases.

## 2. Modelling

Figure 1 shows the two-dimensional model for starved hydrodynamic lubrication. The calculation domain is  $1 \times 1$  mm, and the upper surface is stationary, while the lower surface moves at a constant sliding velocity  $u = 0.1$  m/s. The initial film thickness  $h_s$ , the minimum film thickness  $h_0$  and viscosity  $\mu$  are listed in Table 1. The flow of lubricants in all domains can be characterized by the following equation

$$\frac{\partial}{\partial x} \left( -\frac{h^3}{12\mu} \frac{\partial p}{\partial x} + \frac{\theta h}{2} (v_x + u) \right) + \frac{\partial}{\partial y} \left( -\frac{h^3}{12\mu} \frac{\partial p}{\partial y} + \frac{\theta h}{2} v_y \right) + \frac{\partial(\theta h)}{\partial t} = 0 \quad (1)$$

where saturation variable  $\theta$  is the ratio of the film thickness to the local gap  $h$ , and  $h$  can be represented as

$$h = h_0 + 4\delta x^2/l^2 \quad (2)$$

$v_x$  and  $v_y$  represent the flow velocities of lubricant on the free surface in the x- and y-axis directions. A governing function is proposed to provide a unified description of the lubricant flow on the free surface, which can be written as:

$$v_x = (\frac{2}{\max(2\theta, 1)} - 1)(u - \frac{(\theta h)^2}{2u} \frac{\partial p}{\partial x}) \quad (3)$$

$$v_y = (\frac{2}{\max(2\theta, 1)} - 1)(-\frac{(\theta h)^2}{2\mu} \frac{\partial p}{\partial y}) \quad (4)$$

As the lubricant flows, the lubrication region is divided into two regions, and in the fully-filled region, it has

$$\begin{cases} \theta = 1 \\ p > \max\{p_s, p_{cav}\} \\ v_x = 0; v_y = 0 \end{cases} \quad (5)$$

where  $p_{\text{cav}}$  is the cavitation pressure.  $p_s$  is the pressure at the free surface of lubricant. In the partially-filled region, lubricant separates from the upper surface, with the flow being controlled by the following equations

$$\begin{cases} \theta < 1 \\ p = \max\{p_s, p_{cav}\} \\ v_x = u - \frac{(h\theta)^2}{2\mu} \frac{\partial p}{\partial x} \\ v_y = -\frac{(h\theta)^2}{2\mu} \frac{\partial p}{\partial y} \end{cases} \quad \text{where } p_s = p_{amb} - \gamma \left( \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} \right) \quad (6)$$

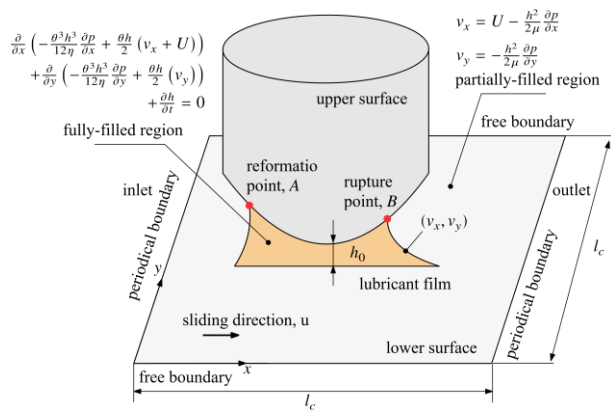


Fig. 1 Modeling of starved lubrication.

Table 1 Simulation parameters

Parameters	Value
Pin height, $\sigma$	32 $\mu\text{m}$
Minimum film thickness, $h_0$	1 $\mu\text{m}$
Initial lubricant supply, $h_s$	2 $\mu\text{m}$
Viscosity, $\mu$	0.35 Pas
Surface tension, $\gamma$	0.03 N/m
Sliding speed, $u$	0.1 m/s
Mesh size	$100 \times 100$
CFL number	0.25

where  $P_{amb}$  represents the ambient pressure, and  $\gamma$  is the surface tension. The CIP-CSL2 method [3] combined with the fractional step technique is employed to solve the aforementioned equations to ensure solution accuracy.

### 3. Results and discussion

Figure 2 illustrates the influence of surface tension on the film distribution. The proposed model successfully reproduces the characteristic butterfly-shaped film profile observed in starved lubrication. When  $\gamma = 0.01$  N/m, no significant accumulation of lubricant is observed on either side of the contact area. Most of the lubricant flows out of the contact area and is distributed along the oil bands. However, as  $\gamma$  increases to 0.2 N/m, the amount of lubricant on the oil bands decreases notably, and the lubricant mainly concentrated on both sides of the contact region. It indicates that, with increasing surface tension, more lubricant is reserved near the contact area.

Further comparison of the film thickness distributions along the x- and y-axes at the contact center reveals that, under low surface tension, the film thickness near the contact area is discontinuous, causing the lubricant prone to separate from the contact area. The results of line y1 show that the inlet meniscus is located very close to the contact center, indicating a severe starvation condition. As surface tension increases, the film distribution near the contact area becomes continuous, and the inlet meniscus shifts upstream. The degree of starvation at the inlet region is alleviated, thus improving the hydrodynamic pressure. This improvement is considered to the enhanced backflow of lubricant from the side oil reservoirs to the contact center due to increased surface tension [4].

Figure 3 shows the effect of lubricant viscosity on the film distribution. At low viscosity, the lubricant is primarily concentrated within the contact area. Film thickness along the x- and y-axes at the contact center show that stable oil reservoirs can form on both sides, allowing more lubricant to flow back to the inlet region and increasing the amount of lubricant entering the contact area. As viscosity increases, the lubricant stored in the reservoirs is gradually reduced due to the enhanced shear forces and the stored lubricant is redistributed to the oil bands outside the contact area. In addition, high viscosity also suppresses the reflow driven by the surface tension [4], resulting in the inlet meniscus moving toward the contact center and increasing the degree of oil starvation.

### 4. Conclusions

In this study, a new starvation model, which incorporates surface tension at the free surface of the lubricant and accounts for flow separation near the outlet region, is proposed. The classical butterfly-shaped film distribution in starved lubrication is predicted. Numerical tests indicate that low lubricant viscosity facilitates the accumulation of lubricant on both sides of the contact area, forming stable oil reservoirs. High surface tension enhances backflow toward the inlet region, thereby alleviating the degree of starvation.

### References

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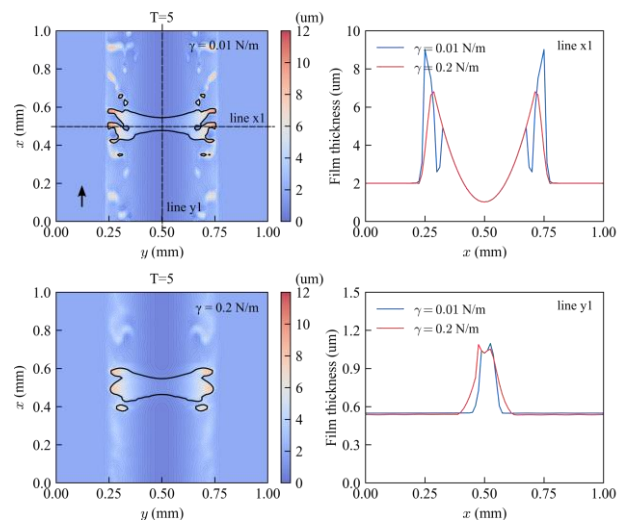


Fig. 2 The effect of surface tension on film distribution.

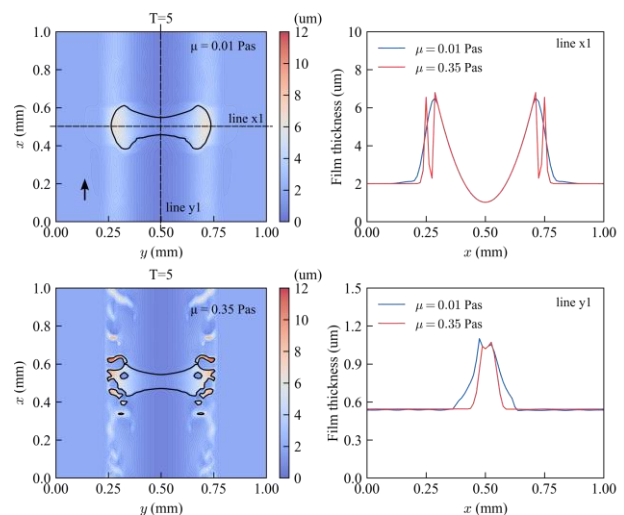


Fig. 3 The effect of lubricant viscosity on film distribution.